

Theoretical Analysis of Uncertainty Visualizations

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ABSTRACT

Although a number of theories and principles have been developed to guide the creation of visualizations, it is not always apparent how to apply the knowledge in these principles. We describe the application of perceptual and cognitive theories for the analysis of uncertainty visualizations. General principles from Bertin, Tufte, and Ware are outlined and then applied to the analysis of eight different uncertainty visualizations. The theories provided a useful framework for analysis of the methods, and provided insights into the strengths and weaknesses of various aspects of the visualizations.

Keywords: uncertainty, visualization, evaluation, perception, Information Visualization, framework

1. INTRODUCTION

The need for visualizing uncertainty along with data now has widespread acceptance. However the task of including the additional uncertainty information into an existing or new visualization while maintaining ease of comprehension for both the data and the uncertainty is not easy. As a result, the visualization of uncertainty is still not standard practice. Various researchers have proposed visualization methods to present uncertainty¹ and some have used HCI methodology to analyze and evaluate their performance.

Recently Johnson and Sanderson called for the development of formal theoretical frameworks for visualizing error and uncertainty.² Before developing new frameworks it is worth examining existing perceptual and cognitive frameworks to better understand them with their strengths and their short comings, and to ensure we are utilizing those frameworks that already exist. With this goal of more fully understanding research in this area, we chose the three commonly cited theoretical approaches and use their principles to analyze eight representative uncertainty visualizations across a wide variety of domains.

A variety of theories and frameworks for analysis of uncertainty and uncertainty visualization are available. In a survey of uncertainty visualization Pang *et al.*³ presented a way of grouping uncertainty methods into the categories: add glyph, add geometry, modify geometry, modify attributes, animation, sonification (not visual), and psychovisual. Previous to that, frameworks for data quality in Geographical Information Systems (GIS) have been proposed by MacEachren⁴ and van der Wal *et al.*⁵ While more recently, Beard and Buttenfield⁶ have created a GIS based framework for error and uncertainty that involves an initial phase of mapping data to error analysis and a second phase mapping to graphical display. While these and other frameworks could be applied and would be useful, at this time we are focusing on the perceptual basics and will only consider the general perceptual and cognitive theories described in the next section.

2. PERCEPTUAL AND COGNITIVE THEORY

From the large number of contributors to perceptual design theory we have chosen a subset of Bertin,⁷ Tufte,⁸ and Ware's⁹ perspectives for our analysis. Other perspectives could have been chosen and may be equally valid, Chambers *et al.*¹⁰ for example. However, since the theories from Bertin, Tufte, and Ware are widely cited, they were deemed to be a good starting point. While we are simply citing Ware's text, we recognize that Ware's collection of theories includes explanations from many cognitive scientists.

Each of this trio of researchers (Bertin, Tufte, Ware) has an extensive set of principles. Therefore to limit the scope we will consider a selection of the trio's perceptual and comprehension driven principles and frameworks.

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This will include Bertin's framework of the plane and retinal variables,⁷ Tufte's theory of data graphics,⁸ and excerpts from Ware's textbook on Information Visualization.⁹

The following overview is included to provide the cognitive context for the specific details that follow. It summarizes the main components from the theory of the Bertin, Tufte, and Ware as used in this paper. Many of these theories and guidelines are represented in all three sources, but from slightly different perspectives.

2.1. Bertin

In Bertin's framework called the Properties of the Graphic System,⁷ he presented eight visual variables. The planar dimensions (x, y) are two of these variables, and for any location on the plane a visible mark can utilize any or all of the six retinal variables: size, value, grain, color, orientation, and shape. These retinal variables were described as being elevated above the plane.

Each of these variables is categorized based on its potential for immediate perceptual group selection, perceptual grouping characteristics, natural perceptual ordering (not learned), ability for quantitative comparisons, and length (the number of discernable elements that can be represented in the set, i.e. cardinality). In terms of perceptual processing speed, a variable is called selective if it can be perceived immediately over the entire plane without considering individual marks sequentially. To be used for selective processing the usable length of any variable must be greatly reduced. If variation in a variable could be ignored so as to consider variation only in other variables, he called the variable associative.

Bertin describes *the plane* as the richest of the variables and is selective, associative, ordered, and quantitative. All the retinal variables must be implanted on the plane. Bertin categorized this implantation as being point, line, or area based. The type of implantation affects the length of the variable. Area implantation raises two issues: the orientation variable can no longer be processed selectively, and the meaning of any variable is read over the entire region of implantation (i.e. quantities must be normalized per unit area or they may be incorrectly read).

Size is the only retinal variable that can be quantitative (allowing ratios of data to be read). It is selective but not associative, and it is ordered. Of note, any variable can be implanted as an area, but when size is implanted as an area, the area itself cannot change size but its constituent points or lines can (see Grain).

Value is the ratio of perceived black to perceived white on a given surface. It is ordered. Bertin's usage is similar to the value in the HSV color model.⁹ Contrasting this, Ware makes a clear distinction in the definitions of luminance, brightness, and lightness from a perceptual context.

Grain is the variation in scale of the constituent parts of a pattern (pattern is texture in French). As grain can be considered a composite of size, it can be ordered on that basis. It is both selective and associative. The length of this variable is based on the size of implantation. Thus making a larger mark allows more steps that can be distinguished.

Color is the variation of two marks with the same value. It is more closely associated with hue variation than saturation. As the pure or monochromatic colors associated with full saturation (not in the HSV sense) do not have equal value, Bertin did not create separate variables for hue and saturation. The color variable has no implicit order, but is selective and associative.

Orientation variation in the angle between marks is the orientation variable. It is associative, but only selective for point and line implantations. It is not ordered. Numerous ways exist to split the 360 degrees of orientation into steps (theoretically infinite). However Bertin states the four steps provide maximal selectivity. To enable the utilization of perceptual sensitivity to parallelism is a main reason for restricting the length to four.

Shape is the most ambiguous variable as it incorporates aspects of size and orientation. It has no ordering. It has infinite length, but is only associative. Its flexible nature allows complex symbolism but this must be learned and therefore is never universally understood.⁷

Bertin's variables can be thought of as one possible set of basis vectors that span the space of 2D visualizations. Expansion beyond the limits of the printed page adds additional visual variables (most obviously depth and time). However 3D visualizations after being projected to 2D can be considered as implantations on the plane, and thus analyzed using the framework.

2.2. Tufte

Tufte has provided principles for graphic excellence and integrity. These are general principles that usually lead to good visualizations. Other aspects of his theory provide optimization rules and design patterns. Tufte has summarized most of his concepts in one complex principle: *graphical excellence* is that which gives to the viewer the greatest number of ideas in the shortest time with the least ink in the smallest space.⁸

In order to create *graphical excellence* Tufte⁸ has specified guidelines such as: avoid distorting what the data shows; encourage the eye to compare the data; present a large amount of data in a small space; reveal multiple levels of detail in the data; and closely integrate statistical and text descriptions into the data. These encourage graphical clarity, precision, and efficiency.⁸ Tufte provides numerous examples of graphical excellence most of which are multivariate.

To promote *graphical integrity* Tufte provides six principles to be followed⁸: one, graphic representations relating to numbers should be directly proportional to the quantities represented; two, clear and detailed text should be used wherever needed to avoid ambiguity; three, show data variation and not design variation; four, money in time series must be adjusted for inflation; five, the number of dimensions used for reading data should not exceed the number of data dimensions being represented (e.g. don't make scalars an area); and six, do not show data out of context. As the name integrity suggests, following these principles avoids deception and misinterpretation.

Data-Ink maximization is a principle that pushes the graphic designer to present the largest amount of data with the least amount of ink. This may have its limits in that one should not keep trying to save ink to the point of breaking of Gestalt Laws¹¹ (to be covered in the next section). Extra ink can be a distraction and take the eye away from seeing the data or making comparisons.

Data density refers to the amount of data elements divided by the area of the graphic. If this is too low it may be reduced in size, or a table may even be more appropriate. Tufte's *small multiples* is a design pattern for comparing data that creates an animation through a series of stills. It states that for a series of graphics the design must remain constant so only the data varies. This should be intuitive, as with scientific analysis we often hold all variables constant except for the one we are trying to investigate.

2.3. Ware

Ware has created a textbook on Information Visualization that draws on numerous researchers' theories (including his own) on visual perception and comprehension.⁹ In general it is grounded in physiological and Cognitive Psychology research rather than the more practically grounded theories of Bertin and Tufte. This research is usually compatible with the previous two's theories and often supports their principles with data from user and electrophysiological studies.

Preattentive Processing — Additional visual variables have been shown to be preattentively processed (Bertin's selective category), some examples are: curvature, spatial grouping, blur, added marks, numerosity, flicker, direction of motion, and stereoscopic depth.

Gestalt Laws — The German Gestalt school of psychology created a set of fundamental laws of pattern perception (Bertin also refers to Gestalt theory).¹¹ Some of these laws describe how properties such as proximity, similarity, continuity, symmetry, closure, connectedness,¹² and relative size have major influence on the perception of patterns. They can be used as design principles in creating visualizations.⁹

Words and Images — Text may often be superior to images for presenting abstract ideas, logic, and conditional information.⁹ Consistent with Tufte's Graphical Excellence principle of integrating text descriptions with a graphic, Ware states that the Gestalt Laws apply (e.g. proximity, or connectedness) when adding text.

Thinking with Visualization — Ware groups and reviews related research dealing with the problem solving aspects of visualization. Memory categories such as iconic memory, long-term memory, and visual working memory are discussed. Theories on eye movement patterns and cognitive data structures are also presented. The implications of these cognitive constraints on problem solving strategies are also reviewed.

3. ANALYSIS OF UNCERTAINTY VISUALIZATIONS

Our methodology borrows from the ideas of heuristic evaluation¹³ as conducted in HCI in which each aspect of a set of heuristics is applied to the interfaces to be analyzed. This section covers eight uncertainty visualizations that were chosen to cover a wide variety of domains. The visualizations also vary from highly data specific to more generally applicable. They will be covered in roughly chronological order. Each in turn will be briefly analyzed using the perceptual theories presented by Bertin,⁷ Tufte,⁸ and Ware.⁹

3.1. Vector Fields

For vector fields we will discuss uncertainty glyphs that Wittenbrink *et al.* introduced in what he called verity visualizations.¹⁴ Uncertainty glyphs represent uncertainty integrated with the data without the use of additional visual variables (color, value, ...). This was done with vector glyphs that holistically show uncertainty in magnitude and orientation. An example vector field using the uncertainty glyphs is shown in Figure 1.

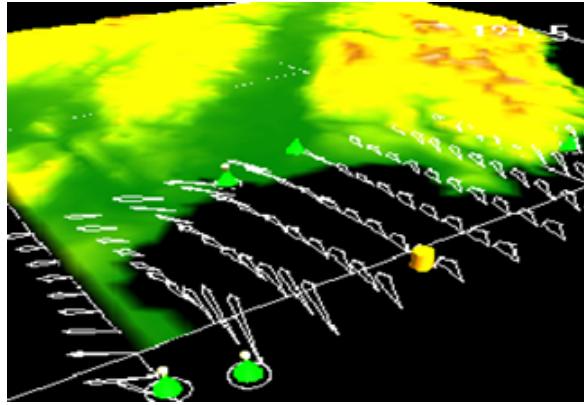


Figure 1. Vector field of glyphs showing uncertainty in orientation and area indicating magnitude.¹⁴ [©1996 IEEE, reprinted with permission]

The authors evaluated their methods using some quantitative measures based on Tufte's data-ink ratio, as well as performing qualitative evaluation with user studies. They found that the mean error for decoding direction with and without uncertainty was not significantly different. This indicates that the addition of their uncertainty visualization was not detrimental to the simpler task which ignored uncertainty. Magnitude decoding with the uncertainty present was found to be practically no different (but statistically different). The uncertainty decoding error was comparable to the respective magnitude and direction error.

Bertin — Wittenbrink *et al.* utilized a combination of Bertin's plane, shape, and size variables for their verity glyph for vector uncertainty. As the plane and size are the only variables Bertin claims may be used quantitatively, this is appropriate usage as the users of their visualization wanted to make quantitative estimates. Showing multivariate data and avoiding using additional visual variables means that the plane and size variables must be overloaded. Decoding these overloaded variables may then be more difficult as the authors discuss.

Bertin's 2D framework may provide insight into potential interpretation problems with 3D viewing. For example, as a vertical line or surface rotates away from the viewer the visible length or area is reduced by the cosine of the angle between the surface normal and the view vector. This directly affects the reading of most variables (to a lesser extent the color and value variables). Therefore the area glyph presented will require more complex cognitive aspects dealing with depth perception to compensate for rotation in 3D.

Tufte — Tufte's principles are discussed and utilized in this paper. They utilize the data-ink maximization theory to design their uncertainty vector glyphs. In integrating the orientation uncertainty into the glyph they found they had to scale the area to the vector magnitude. This was needed as the orientation uncertainty made the glyph larger and area is perceived over length⁸ (length previously being vector magnitude). This treads on Tufte's integrity principle that the number of information carrying dimensions should not exceed the dimensions

of the data (one for magnitude). This type of required trade-off in using these principles is to be expected, but even when not followed the principles provide a warning to potential areas of misinterpretation.

Ware — Ware and Tufte's principle of close integration of text and graphics could be used to provide interactive queries of the glyphs exact values. As the authors were determining how well the new glyphs could be decoded, text was not appropriate, but it could be useful in a final visualization. Gestalt theory also provides a check on the glyph design: symmetry and closure exist with orientation only glyph, but when the magnitude uncertainty is added with an extra leading edge (not illustrated in the figure) it only is perceived as a unit on the basis of proximity. Therefore this perception could become ambiguous when very large magnitude uncertainties exist. A single line from the tip of the arrow to the extra leading edge could provide connectedness to avoid this problem. The trade-off is that this reduces Tufte's data-ink ratio.

3.2. Molecular Structure

Methods for the visualization of molecular positional uncertainty were presented by Rheingans and Joshi.¹⁵ The uncertainty representations used transparency, volume rendering, and iso-surfaces. Traditional ball and stick models were rendered making dynamic portions of the molecule more transparent. In additional methods presented, Gaussian distribution functions representing atoms were first aggregated. These were then either directly volume rendered or iso-surfaces were created based on confidence thresholds (iso-levels). Two example visualizations using the iso-surface method are shown in Figure 2.

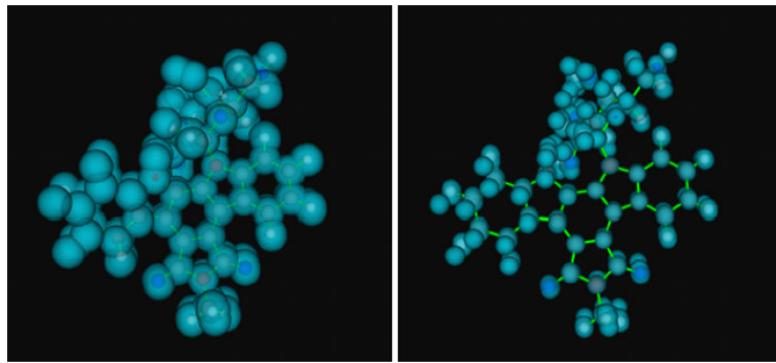


Figure 2. Likelihood iso-surfaces. Left and right images show the same data but with different iso-level values.¹⁵ [©1999 Springer, reprinted with permission]

The authors found that their ball and stick and iso-surface visualizations were flexible enough to provide suitable results for different goals. They conclude that the volume rendering method provided a more holistic representation of the uncertainty. A more rigorous task analysis and performance evaluation was not reported.

Bertin — Transparency performs a blending of the quantities in Bertin's value and color variables as in the limit both are reduced to the background instantiation of these variables. Thus using transparency for uncertainty provides redundant encoding (value and color) of this information and so should be more easily perceived.

Tufte — The data density of these visualizations is high, especially in the volume renderings in which the entire probability distribution is represented. However with the volume rendering identifiable structures became less clear, therefore it could be useful to have the option of integrating text labeling for the atoms, or atom chains (of course layout management may be difficult).

Ware — The authors also compare the transparency effect to motion-blur, and as Ware discusses blur is an additional preattentive (Bertin's selective) visual variable. Interaction with the visualizations was not described, and sadly the lack of information on this aspect is not unique to this paper. The authors briefly mention the fact that using the ball and stick model and controlling the opacity allows the image to be perceptually divided into stable and dynamic regions. It is not clearly stated in this paper whether this could be performed interactively as a dynamic query.¹⁶ Once generated, the iso-surfaces could likely be interactively rendered, but this precludes dynamic query-like behaviour.

3.3. Archaeological Reconstructions

Strothotte *et al.*¹⁷ discuss aspects of non-photorealistic rendering and how they might be applied to representing uncertainty in virtual reconstructions. They show how sketch-like renditions and the use of variable transparency can express the speculative nature of archaeological reconstruction. Figure 3 shows some of their results in which a theoretical reconstruction with various levels of uncertainty is integrated into a photograph of the current excavation site. The authors found that photorealistic detail distracts from the fundamental questions of the domain experts. They conclude that more methods of visualization and interaction are required for expressing the appropriate level of uncertainty. No evaluation of their methods was reported. Earlier related work discusses the software that was used in more detail (AncientVis).¹⁸

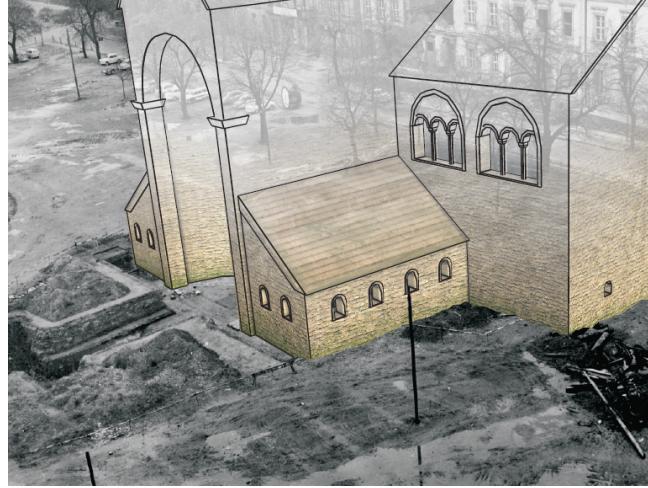


Figure 3. Virtual reconstruction using transparency and line drawings to convey uncertainty.¹⁷ [©1999 Strothotte *et al.*, reprinted with permission]

Bertin — The previous comments from Bertin's perspective on transparency as discussed under molecular visualization are again valid. Bertin states that it is difficult to disregard part of the signifying plane and so an absence of signs indicates absence of data. A line rendering is consistent with this idea, and so is appropriate for the illustration of uncertainty.

Tufte — Graphical integrity is applicable to the goals of this type of visualization. The authors point out that the researchers in this domain are very careful to choose verbal descriptions that convey levels of uncertainty. Photorealistic renderings are only potential interpretations of the archaeological data and so using Tufte's Lie Factor⁸:

$$K_{lie} = \frac{\text{size}(\text{effect}_{graphic})}{\text{size}(\text{effect}_{data})},$$

photorealistic renderings could have potentially huge Lie Factors. Thus portraying the uncertainty is essential to the integrity of the visualization. Line renderings also maximize the data-ink ratio.

Ware — Ware's presentation of various cognitive models for objects is applicable as well. Silhouette and contour information may be key aspects used in forming cognitive models^{19, 20} and so these may be all that is needed to visually express an interpretation. Perceptual theories more directly related to non-photorealistic rendering can be found in Strothotte's textbook.²¹

3.4. 2D Stochastic Simulation

Various methods for visualizing 2D probability distributions have been presented by Kao *et al.*²² With their data at each pixel (cell) probability density functions exist based on the different realizations from multiple stochastic simulations. They claim that the spread of a distribution is the most obvious way to summarize uncertainty.

Kao *et al.* provided visual renderings of statistical measures such as mean, median, and quantiles, on a per-pixel basis. These visualizations used color, surface, and spatial bar charts for presenting various statistical measures and let the users choose the mapping. An example pixel based analysis view is illustrated in Figure 4 with the user selected mapping detailed in the annotation. To reduce clutter they provided thresholds for the filtering of insignificant uncertainty representations. A feature-wise analysis tool based on clumps (similarly behaved region) was also described. Kao *et al.* also present a histogram cube to visualize the distribution of the data. Each slice represents a bin for which the pixels contain the bin count at that location. They found it was useful for understanding the modality of the distributions. The authors conclude that their visualizations were useful, but no formal evaluation was done. Future user studies were proposed following the initial user feedback during the design and development phases. Navigation techniques for the 3D view shown in Figure 4 were not described and this would be important due to the amount of detail present.

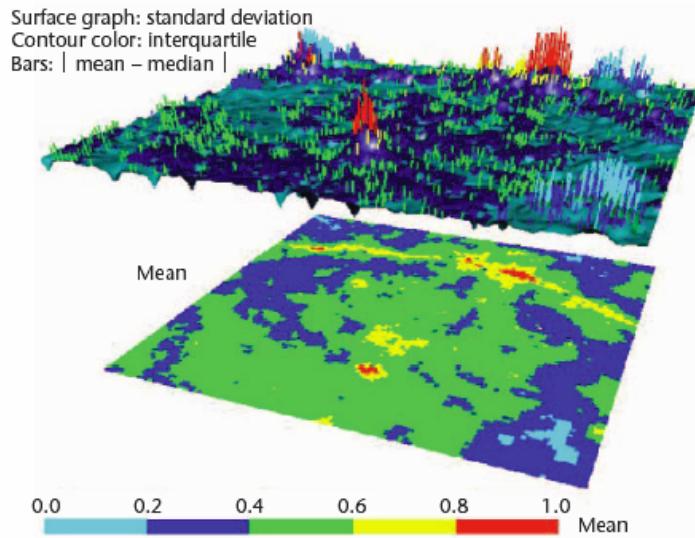


Figure 4. Pixel-wise analysis of data distributions.²² [©2001 IEEE, reprinted with permission]

Bertin — For spatial data the use of a colormap that varies in both value and color still leaves size, grain, orientation, and shape variables for additional information. These variables may be easier to cognitively integrate than the additional 3D surface (that uses the plane, size, and color variables).

Tufte — It may be worth considering Tufte's integrity principle: do not show data out of context. Complex classifications that do not reflect topographical or other known spatial distributions will be difficult to cognitively integrate into the correct spatial context. Text annotations or symbol landmarks could help with this integration by labeling extrema (on low pass filtered data) and showing these same landmarks on an adjacent terrain map.

Ware — As can be seen in Figure 4 the implementation allows the two representations to be viewed simultaneously. While the small multiples design pattern is not directly applicable, if the goal is to understand relationships then orthographic projection would maintain size consistency and simplify cognitive integration. Ware discusses various issues relating to context and cognitive integration. Numerous other aspects relating to navigation and maps would be applicable.

3.5. Grid-based Annotation Lines

Cedilnik and Rheingans have presented procedural rendering of annotation overlays that indicate uncertainty.²³ The authors show how procedural variation of width, sharpness, noise, and amplitude modulation can indicate uncertainty. The illustration of uncertainty only on the annotation (grid) lines allows the data to remain largely unobscured, as can be seen in Figure 5. The authors state that their method preserves perceptibility across various levels of uncertainty. No formal evaluation was reported.

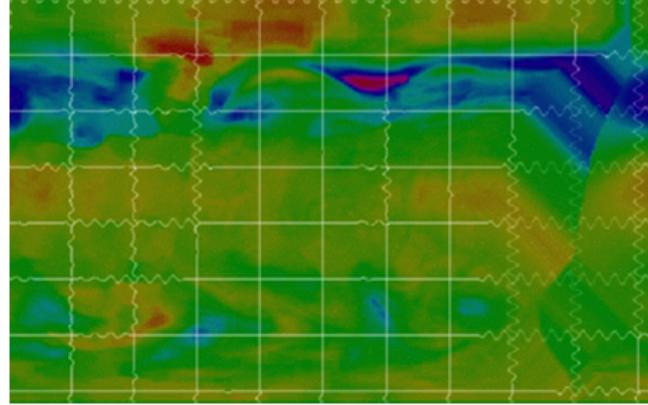


Figure 5. Procedural grid overlay with sine wave amplitude modulation indicating uncertainty of data at that location.²³ [©2000 IEEE, reprinted with permission]

Bertin — Bertin's variables of the plane are mainly used for the amplitude modulation although it crosses into use of grain. The size and value variables are used for the width and sharpness techniques. The noise-based annotation is more ambiguous as it has aspects of size, value, and grain. From this one would expect the largest number of levels of uncertainty would be discernable with the amplitude modulation.

Tufte — Tufte's data-ink maximization rule would suggest that the amplitude modulation would also be the best of their methods. The data-ink ratio in the noise based method could easily be increased by only showing random points along the maximum displacement, but this would violate the authors' energy (perceptibility) conservation scheme.

Ware — The authors state for all methods they attempt to perceptually normalize the amount of energy present at every place. For energy conservation they integrate an annotation intensity value for normalization. However they map it to saturation from HSV space, which Ware points out, is only crudely linear in perceptual space. This approximation is a trade-off that must be made against run-time speed. Therefore they conserve perceptual energy by trying to transfer perceptibility from the reading of Bertin's value and color variables to the size variable (assuming Bertin's line implantations). It would be interesting to more formally evaluate how well this works. Ware also points out that size of an object impacts the perception of color so this is a difficult perceptual balancing act.

3.6. Particle Movement

Methods for the visualization of uncertain 2D and 3D particle movement over time were presented by Lodha *et al.*²⁴ Size (spheres), transparency, and color were used to visualize the resultant probability distributions. They found that the resulting visualizations could often be categorized by form as can be seen in the Figure 6. The addition of color to transparency was found to better delineate high probability regions. The authors found that their algorithm and subsequent visualizations were useful for understanding probabilistic movement and distributions. No formal evaluations were discussed.

Bertin — The authors found that the use of transparency and color more clearly showed the high density regions in the center. Again the addition of the color variable adds length beyond the range of perceptual steps available from transparency (a value and color hybrid).

Tufte — A rule of graphical excellence suggests close integration of statistical and verbal descriptions of the data. It would likely be beneficial to add numeric, textual, or graphic (principle component axes) annotation directly on the visualization. This would be especially true in 3D.

Ware — Again Gestalt theory¹¹ comes into play for the perception of shapes. Analysis of these laws may provide the validation that the shape being perceived is capturing all the relevant aspects of these probability distributions. The authors found that adding color helped to understand regions in the uncertainty. Even before

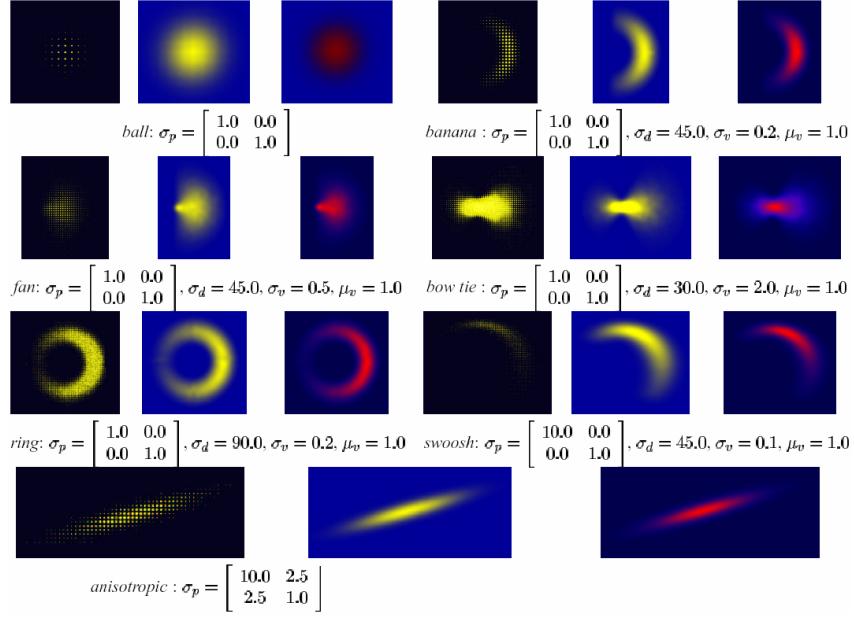


Figure 6. Seven different 2D probability clouds. Three different representations for each cloud shown in subimages from left to right: spheres, transparency, and transparency and color (the color differentiates high opacity into 2 regions). Cloud forms are assigned shape names such as ball, banana, fan....²⁴ [©2002 Lodha *et al.*, reprinted with permission]

adding pseudo-colors, transparency effectively acted as a color saturation variable due to the background color. Ware presents various research that may be useful in this area. Ware²⁵ has shown that errors in reading gray and saturation scales can be as large as 20% of the scale. However gray scales (value) do provide the highest spatial frequency sensitivity.⁹ Other color sequences could be used to more clearly delineate more than two probability regions.

3.7. Air Traffic Flow Decision Support

In quite a different domain, Masalonis *et al.* discuss the visualization of uncertainty in air traffic flow management.²⁶ In this domain the user's task is making decisions based on probabilistic alert levels. They have a discrete probability density function representation of the uncertainty and consider showing it in a more detailed drill-down view. The authors performed a qualitative task analysis for which they carried out a user study. The user study covered various aspects of the cognitive issues related to operational needs of the uncertainty display. They then proposed multiple views that have various levels of detail and meta-data related to the uncertainty modeling. One of these views relating to alert likelihood monitoring is shown in Figure 7. As work was still in the design phase they did not get to the point of evaluating their proposed designs or prototypes.

Bertin — As Bertin's color variable is used to show the probability of an alert (3 levels are used: green, yellow, and red), using this variable there is enough length for even more alert levels. Similarly color saturation (and value) could provide more levels within each of the three probability regions if needed.

Tufte — On the overall display Tufte's data density measure appears to be quite low. This might suggest that more data could be presented in the Overview Display (not shown in figure). The numbers could possibly be removed completely if more color levels were utilized. This might also change the scanning strategy if users tried to anticipate the alert changes using the numbers (i.e. green changing to yellow).

Ware — The choice of pure red and green colors excludes a large number of color blind people from performing the task. Around 10% of the general male population and 1% of the female population are color deficient,⁹ with red-green being the most common. Ware also discusses that large regions of color should use low saturated colors to avoid visual stress. Therefore depending on the size of the display (a prototype had an 11x12 matrix of cells

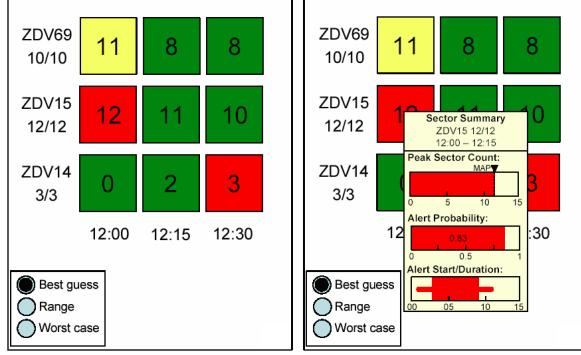


Figure 7. Mock-up of alert display with colors (top left “11” square is yellow, “12” & “3” red, & the rest are green) indicating probability of exceeding an alert threshold. Left to right images illustrate result of mouse rollover, or hover query.²⁶ [©2004 MITRE Corporation, reprinted with permission]

similar to those in the mockup in Figure 7) the green and yellow colors should be very low saturation. Another of Ware’s color design guidelines is that a text to background luminance ratio of 10:1 is preferred (3:1 is the ISO 9241 part 3 minimum recommendation⁹). The luminance ratio of the black text on green would also increase after replacement with lower saturation colors.

As this visualization involves a visual monitoring task, Ware’s coverage of attention and scanning strategies theory such as Wickens²⁷ should be useful. Motion and flicker are visual variables that extend further in the user’s useful field of view. Depending on the final display size they could be used to help avoid missing significant uncertainty changes.⁹ Charbonnell *et al.*²⁸ and Sheridan²⁹ have proposed that monitoring behaviour is controlled by growth of uncertainty in a channel and the cost of sampling a channel. Prolonged viewing in the case of monitoring may also lead to over polling of low frequency data.³⁰ Implications from other monitoring research are also discussed.^{31,32} Interaction issues surrounding the use of hover queries are summarized by Ware, such as Rutkowski’s principle of transparency³³ in which the tool itself disappears and one can focus single-mindedly on the task. All this suggests that there might be alternative visualizations to help with the monitoring nature of the task.

3.8. Surfaces

Grigoryan and Rheingans have shown how points and lines can be used to represent uncertainty in a 3D surfaces position.³⁴ Starting with a surface segmented from medical data, a large number of points are pseudo-randomly displaced along the surface normals according to the uncertainty. An example of their visualization method using a tumour segmented from an MRI scan is provided in Figure 8. Lines can also be drawn from the zero displacement surface to the point. Their method supports both a uniform distribution or when available a probability density function (PDF) based distribution. Results from a preliminary evaluation comparing their point displacement to a pseudo-color representation were reported. The task was determining if an object was within a specific error margin around a surface. The point-based scheme showed an average increase in accuracy of 20% ($p < 0.01$) and made judgments faster, although this had lower statistical significance ($p < 0.1$). Subjective ratings of ease, confidence, and satisfaction were all also higher for the point-based representation ($p < 0.01$) over pseudo-coloring.

Bertin — The displacement of points in the plane and the use of size provide the quantitative aspects required for this application domain. They also tried using a neutral color and transparency to encode uncertainty. As the length of the color variable is small compared to the plane and size it was appropriate that it was only used as binary threshold to switch from a specific color to a neutral color (e.g. gray) based on the uncertainty.

The accuracy and speed results from the user testing are predicted from Bertin’s theory. As the objects’ spatial extents are represented in the plane, if the uncertainty is also represented in the plane direct reading can provide the query result (the plane also allows quantitative reading). This leads to a relevant point from Bertin

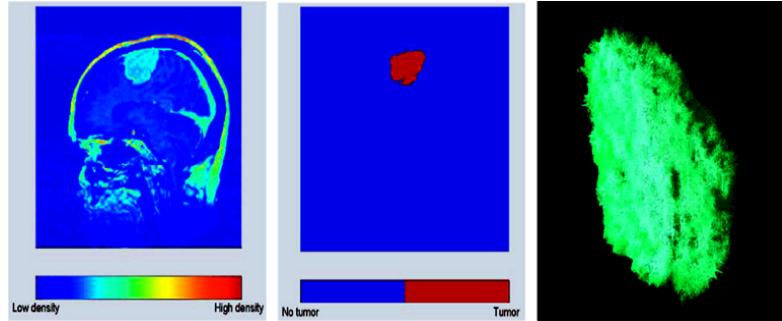


Figure 8. Images from left to right: MRI scan, segmented tumour, tumour surface in which points are displaced along the surface normal based on an uncertainty distribution.³⁴ [©2004 IEEE, reprinted with permission]

that certainty of the uniformity of the plane entails a presumption of uniformity in the conventions adopted within the signifying space.⁷ The authors describe a user controlled scaling factor for the displacement, but if this does not match the domain of the PDF it violates the uniformity in convention Bertin describes. This violation would occur as the point displacements and the zero displacement surfaces are both represented in the plane, and the non-uniformity could lead to misinterpretation.

Tufte — Graphical integrity is clearly in question with the presentation of an uncertain surface as a clean, precise, polygonal surface. This is why the authors have attempted to build a truer representation. The user controlled scaling factor mentioned in the Bertin section could also relate to a potential increase in the Lie Factor. As the rendering was interactive it could benefit from additional text annotation, perhaps based on a user controlled probe. Another guideline for excellence that might be applied is to reveal the data on several levels of detail. As the point based display is full of fine detail the user should be able to toggle it with the uncertainty free surface, or provide a mouse draggable inset for this level of detail.

Ware — While the use of transparency has good properties for representing uncertainty, Ware reviews its limitations. The use of lighting along with color variation (the more general definition of color) may also be problematic as this overloads the value and color variables to the point of potential misinterpretation. Value (luminance) therefore should not be part of the color variation, and this was not explicitly stated by the authors. As the visualization was interactive it is assumed the user could manipulate the viewpoint or object. Therefore understanding the context of the data is important. In cases such as the tumour dataset, understanding the uncertainty in relation to the surrounding tissue is of vital importance. This could be done by merging the visualization with an interactive slice planes from the original volumetric scan data.

4. CONCLUSIONS

The theories provided by Bertin, Tufte, and Ware were relevant to all of these uncertainty visualizations. It is important to note that these visualizations were chosen as a representative sample of uncertainty visualization and this choice was not in any way based on the potential applicability of these theories. Most of the visualizations did not mention these theories, and Wittenbrink *et al.*¹⁴ was the exception which explicitly utilized Tufte's and others theories to refine and analyze their solution.

Analysis using these theories can be considered as a form of heuristic evaluation.^{13,35} We summarize a subset of the applied theory, in the form of possible heuristics, in Table 1. These heuristics are over simplifications, but the application of them may raise important issues. The relevance count in the table is only provided to summarize applicability for the eight visualizations reviewed and not intended to imply the relative generality. Future work could apply these perceptual and cognitive heuristics to more visualizations to better determine their generality and usefulness.

Often the authors stated that future work would be in evaluating their new methods in the form of user studies, and Wittenbrink *et al.*¹⁴ and Grigoryan and Rheingans³⁴ did perform and report their evaluation results.

Table 1. Potential Heuristics. The “Heuristic” column presents the simplified forms of the theory. The “Relevant” column indicates the total number of visualizations for which the heuristic was pertinent.

Heuristic	Source	Relevant (n/8)
Ensure visual variable has sufficient length	Bertin & Ware	7
Preserve data to graphic dimensionality	Tufte & Bertin	2
Put the most data in the least space	Tufte	2
Provide multiple levels of detail	Tufte & Ware	2
Remove the extraneous (ink)	Tufte	4
Consider Gestalt Laws	Ware	2
Integrate text wherever relevant	Tufte & Ware	6
Don't expect a reading order from color	Bertin & Ware	1
Color perception varies with size of colored item	Ware & Bertin	2
Local contrast affects color & gray perception	Ware	2
Consider people with color blindness	Ware	2
Preattentive benefits increase with field of view	Bertin & Ware	3
Quantitative assessment [†] requires position or size variation	Bertin	4

[†]Perceiving an accurate approximation of the ratio between two signs or grouping of homogeneous signs.⁷

Masalonis *et al.*'s²⁶ research was also unique as it reported analysis from a user study done during the initial stages (task analysis & design) of creating a visualization.

The amount of work involved in evaluation often forces the two part presentation of research: development and then evaluation. Obviously it is the second part that may not get done, and even when performed may not make it into publication. This work suggests that potentially more light-weight evaluations, in a manner similar to what we have done here, could more often be included in current work. The need for greater application of human factors research to visualization has also recently been noted by Tory and Möller.³⁶ Following this lead further, we should examine the Cognitive Psychology literature dealing with uncertainty and probabilistic reasoning (such as Kahneman *et al.*,³⁷ Gilovich *et al.*,³⁸ Sloman *et al.*,³⁹ Kirschenbaum and Arruda,⁴⁰ and Finger and Bisantz⁴¹); as the uncertainty if correctly understood must then be integrated into a decision process.⁴² This decision process adds cognitive load, which may restrict the resources available for the visualization process.

Recent research continues to develop new frameworks. Amar and Stasko produced a knowledge task-based framework for design and evaluation of information visualizations.⁴³ One component of their category of rationale-based tasks was to expose uncertainty. Thomson *et al.* have constructed an in-depth typology for visualizing uncertainty.⁴⁴ They consider contributions from Pang *et al.*'s classification³ and Gershon's high-level taxonomy⁴⁵ of uncertainty. Their general taxonomy for geospatially referenced data is instantiated based on task (e.g. intelligence analysis). The aforementioned framework and general taxonomy should complement the lower level perceptual and cognitive theories used for analysis in this paper.

Uncertainty visualization should not be considered unique; we expect the theories would be similarly relevant to most other visualization problems. While a few of the uncertainty paper authors discussed and applied the theories, it appears that they have been under utilized. We would even suggest that detailed analysis from their perspectives should be more strongly influencing the work in the field of visualization.

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